mountain, that had been forming within it for ſome years, and was riſen above the ſides; and throwing up, by violent exploſions, immenſe quantities of ſtones, lava, aſhes, and fire. At night the flames burſt out with greater vehemence, the exploſions were more frequent and horrible, and our houſes ſhook continually. Many fled to Naples, and the boldeſt perſons trembled. For my own part, I reſolved to abide the event here at Portici on account of my family, conſiſting of eight children, and a very weak and aged mother, whoſe life muſt have been loſt by a removal in ſuch circumſtances, and ſo rigorous a ſeaſon. But it pleaſed God to preſerve us; for the mountain having vented itſelf that night and the ſucceeding day, is ſince become calm, and throws out only a few aſhes.

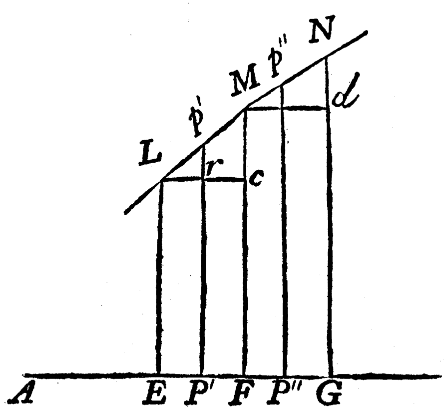
LXXXV. *A further Attempt to facilitate the Reſolution of Iſoperimetrical Problems. By Mr.* Thomas Simpſon, *F. R. S.*

Read April 13. 1758.

ABOUT three years ago I had the honour to lay before the Royal Society the inveſtigation of a general rule for the reſolution of iſoperimetrical problems of that kind, wherein one, only, of the two indeterminate quantities enters along with the fluxions, into the equations expreſſing the conditions of the problem. Under which kind are included the determination of the greateſt figures under given bounds, lines of the ſwifteſt deſcent, ſolids of the leaſt reſiſtance, with innumerable other caſes. But altho' caſes of this ſort do, indeed, moſt frequently occur, and have therefore been chiefly attended to by mathematicians, others may nevertheleſs be propoſed, ſuch as actually ariſe in inquiries into nature, wherein *both* the flowing quantities, together with their fluxions, are jointly concerned. The inveſtigation of a *rule* for the reſolution of theſe, is what I ſhall in this paper attempt, by means of the following

GENERAL PROPOSITION.

Let Q, R, S, T, &c. *repreſent any variable quantities, expreſſed in terms of* x *and* y *(with given coefficients), and let* q, r, s, t, &c. *denote as many other quantities, expreſſed in terms of* $\dot{x}$ *and* $\dot{y}$; *It is propoſed to find an equation for the relation of* x *and* y, *ſo that the fluent of* $Qq + Rr + Ss + Tt$, &c. *correſponding to a given value of* x *(or* y*), may be a* maximum *or* minimum.



Let

Let AE, AF, and AG, denote any three values of the quantity x, having indefinitely ſmall *equi-differences* EF, FG; and let EL, FM, and GN, (perpendicular to AG) be the reſpective values of y, correſponding thereto; and, ſuppoſing $EF (=FG=\dot{x})$ to be denoted by e, let cM and dN (the ſucceſſive values of $\dot{y}$) be repreſented by u and w. Moreover, ſuppoſing $P'p'$ and $P''p''$ to be ordinates at the middle points P' P'', between E, F and F, G, let the former $(P'p')$ be denoted α, and the latter $(P''p'')$ by β; putting $AP'=a$ and $AP''=b$. Then, if a and α (the mean values of x and y, between the ordinates EL and FM) be ſuppoſed to be ſubſtituted for x and y, in the given quantity $\mathfrak{Q}q+Rr+Ss+Tt$, &c. and if, inſtead of $\dot{x}$ and $\dot{y}$, their equals e and u be alſo ſubſtituted, and the ſaid (given) quantity, after ſuch ſubſtitution, be denoted by $\mathfrak{Q}'q'+R'r'+S's'+T't'$, &c. it is then evident, that this quantity $\mathfrak{Q}'q'+R'r'+S's'+T't'$, &c. will expreſs ſo much of the whole required fluent, as is comprehended between the ordinates EL and FM, or as anſwers to an increaſe of EF in the value of x. And thus, if b and β be conceived to be wrote for x and y, e for $\dot{x}$, and w for $\dot{y}$, and the quantity reſulting be denoted by $\mathfrak{Q}''q''+R''r''+S''s''+T''t''$, &c. this quantity will, in like manner, expreſs the part of the required fluent correſponding to the interval FG. Whence that part anſwering to the interval EG will conſequently be equal to $\mathfrak{Q}'q'+R'r'$ &c. $+\mathfrak{Q}''q''+R''r''$ &c. But it is

 manifeſt,

manifest, that the whole required fluent cannot be a *maximum* or *minimum*, unless this part, supposing the bounding ordinates EL, GN to remain the same, is also a *maximum* or *minimum*. Hence, in order to determine the fluxion of this expression ($Q'q' + R'r'$ &c. $Q''q'' + R''r''$ &c.) which must, of consequence, be equal to nothing, let the fluxions of Q' and q' (taking α and u as variable) be denoted by $\bar{Q}\dot{\alpha}$ and $\bar{q}\dot{u}$; also let $\bar{R}\dot{\alpha}$ and $\bar{r}u$ denote the respective fluxions of R' and r'; and let, in like manner, the fluxions of Q'', q'', R'', r'', &c. be represented by $\bar{\bar{Q}}\dot{\beta}$, $\bar{\bar{q}}\dot{w}$, $\bar{\bar{R}}\dot{\beta}$, $\bar{\bar{r}}\dot{w}$, &c. respectively. Then, by the common rule for finding the fluxion of a rectangle, the fluxion of our whole expression ($Q'q' + R'r'$ &c. $+ Q''q'' + R''r''$ &c.) will be given equal to $Q'\bar{q}\dot{u} + q'\bar{Q}\dot{\alpha} + R'\bar{r}\dot{u} + r'\bar{R}\dot{\alpha}$ &c. $+ Q''\bar{\bar{q}}\dot{w} + q''\bar{\bar{Q}}\dot{\beta} + R''\bar{\bar{r}}\dot{w} + r''\bar{\bar{R}}\dot{\beta}$ &c. $= 0$.

But $u + w$ being $= GN - EL$, and $\beta - a = \frac{GN - EL}{2}$ (a constant quantity), we therefore have $\dot{w} = -\dot{u}$, and $\dot{\beta} = \dot{\alpha}$: also u being $(= 2rp') = 2\alpha - 2EL$, thence will $\dot{u} = 2\dot{\alpha}$: which values being substituted above, our equation, after the whole is divided by $\dot{\alpha}$, will become

$$2Q'\bar{q} + q'\bar{Q} + 2R'\bar{r} + r'\bar{R}, \text{ \&c.} - 2Q''\bar{\bar{q}} + q''\bar{\bar{Q}} - 2R''\bar{\bar{r}} + r'\bar{\bar{R}}, \text{ \&c.} = 0;$$

$$\text{or, } Q''\bar{\bar{q}} - Q'\bar{q} + R''\bar{\bar{r}} - R'\bar{r} \text{ \&c.} = \frac{q'\bar{Q} + q''\bar{\bar{Q}}}{2} + \frac{r'\bar{R} + r''\bar{\bar{R}}}{2}, \text{ \&c.}$$

But

But $Q''\bar{\bar{q}} - Q'\dot{\bar{q}}$, the excess of $Q''\bar{\bar{q}}$ above $Q'\dot{\bar{q}}$, is the increment or fluxion (answering to the increment, or fluxion, $\dot{x}$) arising by substituting b for a, β for α, and w for u. Moreover, with regard to the quantities on the other side of the equation, it is plain, seeing the difference of $q'\bar{Q}$ and $q''\bar{\bar{Q}}$ is indefinitely little in comparison of their sum, that $q'\bar{Q}$ may be substituted in the room of $\frac{q'\bar{Q} + q''\bar{\bar{Q}}}{2}$, &c. which being done, our equation will stand thus:

$$Flux.\ Q'\bar{q} + R'\bar{r}\ \&c. = q'\bar{Q} + r'\bar{R}\ \&c.$$

But $q'\bar{Q} + r'\bar{R}$ &c. represents (by the preceding notation) the fluxion of $q'Q' + r'R'$ &c. (or of $Qq + Rr$ &c.) arising by substituting α for y, making α alone variable, and casting off $\dot{\alpha}$. If, therefore, that fluxion be denoted by $\dot{v}$, we shall have *flux.* $Q'\bar{q} + R'\bar{r}$ &c. $= \dot{v}$, and consequently $Q'\bar{q} + R'\bar{r}$ &c. $= v$. But $Q'\bar{q} + R'\bar{r}$ &c. (by the same notation) appears to be the fluxion of $Q'q' + R'r'$ &c. (or of $Qq + Rr$ &c.) arising by substituting u for $\dot{y}$, making u alone variable, and casting off $\dot{u}$. Whence the following

General Rule.

Take the fluxion of the given expression (whose fluent is required to be a maximum *or* minimum) *making* $\dot{y}$ *alone variable; and, having divided by* $\ddot{y}$, *let the quotient be denoted by* v: *Then take, again, the fluxion of the same expression, making* y *alone variable, which divide by* $\dot{y}$; *and then this last quotient will be* $= \dot{v}$.

When

When $\dot{y}$ is not found in the quantity given, υ will then be $= 0$; and, consequently, the expression for $\dot{\upsilon}$, equal to nothing also. But if y be absent, then will $\dot{\upsilon} = 0$, and consequently the value of $\upsilon =$ a constant quantity. It is also easy to comprehend, that, instead of $\dot{y}$ and y, $\dot{x}$ and x may be made successively variable. Moreover, should the case to be resolved be confined to other restrictions, besides that of the *maximum* or *minimum*, such as, having a certain number of other fluents, at the same time, equal to given quantities, still the same method of solution may be applied, and that with equal advantage, if from the particular expressions exhibiting all the several conditions, one general expression composed of them all, with unknown (but determinate) coefficients, be made use of.

In order to render this matter quite clear, let A, B, C, D, &c. be supposed to represent any quantities expressed in terms of x, y, and their fluxions, and let it be required to determine the relation of x and y, so that the fluent of $A\dot{x}$ shall be a *maximum*, or *minimum*, when the cotemporary fluents of $B\dot{x}$, $C\dot{x}$, $D\dot{x}$, &c. are, all of them, equal to given quantities.

It is evident, in the first place, that the fluent of $A\dot{x} + bB\dot{x} + cC\dot{x} + dD\dot{x}$, &c. ($b$, c, d, &c. being any constant quantities whatever) must be a *maximum*, or *minimum*, in the proposed circumstance: and, if the relation of x and y be determined *(by the rule)*, so as to answer this single condition (under all

possible

poſſible values of b, c, d, &c.) it will alſo appear evident, that ſuch relation will likewiſe anſwer and include all the other conditions propounded. For, there being in the general expreſſion, thus derived, as many unknown quantities b, c, d, &c. (to be determined) as there are equations, by making the fluents of $B\dot{x}$, $C\dot{x}$, $D\dot{x}$, &c. equal to the values given; thoſe quantities may be ſo aſſigned, or conceived to be ſuch, as to anſwer all the conditions of the ſaid equations. And then, to ſee clearly that the fluent of the firſt expreſſion, $A\dot{x}$, cannot be greater than ariſes from hence (other things remaining the ſame) let there be ſuppoſed ſome other different relation of x and y, whereby the conditions of all the other fluents of $B\dot{x}$, $C\dot{x}$, $D\dot{x}$, &c. can be fulfilled; and let, *if poſſible*, this new relation give a greater fluent of $A\dot{x}$ than the relation above aſſigned. Then, becauſe the fluents $bB\dot{x}$, $cC\dot{x}$, $dD\dot{x}$, &c. are given, and the ſame in both caſes, it follows, according to this ſuppoſition, that this new relation muſt give a greater fluent of $A\dot{x} + bB\dot{x} + cC\dot{x} + dD\dot{x}$, &c. (under all poſſible values of b, c, d, &c.) than the former relation gives: *which is impoſſible*; becauſe (whatever values are aſſigned to b, c, d, &c.) *that* fluent will, it is demonſtrated, be the greateſt poſſible, when the relation of x and y is that above determined, by the General Rule.

To exemplify, now, by a particular caſe, the method of operation above pointed out, let there be proposed the fluxionary quantity $\frac{x^n y^m \dot{y}^p}{\dot{x}^{p-1}}$; wherein the relation of x and y is so required, that the fluent, corresponding to given values of x and y, shall be a *maximum*, or *minimum*. Here, by taking the fluxion, making $\dot{y}$ alone variable (*according to the rule*) and dividing by $\ddot{y}$, we shall have $\frac{px^n y^m \dot{y}^{p-1}}{\dot{x}^{p-1}} = v$. And, by taking the fluxion a second time, making y alone variable, and dividing by $\dot{y}$, will be had $\frac{mx^n y^{m-1} \dot{y}^p}{\dot{x}^{p-1}} = \dot{v}$. Now from these equations to exterminate v, let the latter be divided by the former; so shall $\frac{m\dot{y}}{py} = \frac{\dot{v}}{v}$; and therefore $ay^{\frac{m}{p}} = v$ (a being a constant quantity). From whence $y^{\frac{m}{p}}\dot{y} = \overline{\frac{a}{p}}\Big|^{\frac{1}{p-1}} \times \dot{x}x^{-\frac{n}{p-1}}$; and consequently $\frac{p}{m+p} \times y^{\frac{m+p}{p}} = \overline{\frac{a}{p}}\Big|^{\frac{1}{p-1}} \times \frac{p-1}{p-n-1} \times x^{\frac{p-n-1}{p-1}}$.

Let there be now proposed the two fluxions $x^n y^m \dot{x}$ and $x^p y^q \dot{y}$, the fluent of the former being required to be a *maximum*, or *minimum*, and that of the latter, at the same time, equal to a given quantity. Then the latter, with the general coefficient b prefixed, being joined to the former, we shall here have $x^n y^m \dot{x} + bx^p y^q \dot{y}$. From whence, by proceeding as before, $bx^p y^q = v$, and $mx^n y^{m-1} \dot{x} + qbx^p y^{q-1} \dot{y} = \dot{v}$.

From

From the former of which equations, by taking the fluxions on both ſides, will be had $pbx^{p-1}y^{q}\dot{x} + qbx^{p}y^{q-1}\dot{y}\ (=\dot{v}) = mx^{n}y^{m-1}\dot{x} + qbx^{p}y^{q-1}\dot{y}$. Whence $pbx^{p-1}y^{q} = mx^{n}y^{m-1}$; and therefore $pby^{q-m+1} = mx^{n-p+1}$. And in the ſame manner proper equations, to expreſs the relation of x and y, may be derived, in any other caſe, and under any number of limitations.

LXXXVI. *Obſervations on the* Alga Marina latifolia; *The Sea Alga with broad Leaves. By* John Andrew Peyſſonel, *M.D. F.R.S. Tranſlated from the* French.

Read April 13. 1758.

HAVING caſt anchor at Verdun, the road at the entrance of the river of Bourdeaux, I was fiſhing with a kind of drag-net upon a bank of ſand, which was very fine and muddy. We collected a number of ſea-plants, and among them the great broad-leaved Alga, which I did not know: and as the root or pedicle of this plant appeared to be very particular, I obſerved it with attention. The following is its deſcription, and the detail of my obſervations.

From a pedicle, which is ſometimes flat, and ſometimes round (for they vary in theſe plants, and might be about three lines in diameter, and an inch high, of a blackiſh colour, and coriaceous ſubſtance, approaching to the nature of the bodies of lithophyta), a ſingle